Journal of Umm Al-Qura University for Applied Science

journal homepage: https://uqu.edu.sa/en/jas/87823

The Vibration of a Viscothermoelastic Gold Nanobeam Induced by Different Types of Thermal Loading

Eman A. N. Al-Lehaibi [a].

[a] *Mathematics Department, Al-Lith University College, Umm Al-Qura University, Al-Lith, Saudi Arabia Arabia.*

A R T I C L E I N F O

Article History:
Submission date: 11/10/2019
Accepted date: 12/4/2020

Keywords:
Vibration; Viscothermoelastic, Golden Nanobeam, Golden, Ramp-type heat, Harmonic heat, Fourier law.

A B S T R A C T

In the present work, the numerical solutions have been carried out for viscothermoelastic homogeneous isotropic nanobeam. A generalized model of one relaxation time has been used, under simply supported conditions for fixed aspect ratios. Laplace transform has been applied for the governing equations. The inverse of the Laplace transform has been calculated by applying the Tzou method. The numerical results have been validated for a viscothermoelastic rectangular nanobeam of gold as a particular case when it is subjected to ramp-type and harmonic-type heating. The numerical results have been illustrated in figures to stand on the impact of the viscothermoelastic parameters, the ramping heat parameter, and the harmonic heat parameter on all the studied functions. The viscothermoelastic parameters, the ramping heat parameter, and the harmonic heat parameter have significant effects on the temperature increment, the lateral vibration, the deformation, the stress, and the stress-strain energy distributions.

1. Introduction

Tzou constructed and studied the heat conduction by using different mathematical models such as dual-phase lag (DPL) [1,2]. The temperature gradient and heat flux have been considered through this model. Many authors have used this model to solve heat transfer problems [3-8]. The coupled theory of thermoelasticity is the kind of heat conduction based on the equation of motion, and the equation of energy conservation, depending on Fourier’s law of heat conduction [9-12]. Lord and Shulman modified the classical Fourier’s law of heat conduction by inserting the lag time (relaxation time) for an isotropic case [13]. Within this model, the heat conduction law has been modified to include the heat flux as an unknown function with its time derivative, which is called Cattaneo’s law (non-Fourier) of heat conduction. The heat equation is a hyperbolic type in this theory, which eliminates the defect of infinite speed propagation of the thermal wave [14]. Many mathematical models and applications based on micro and nano-electromechanical beam resonators have been solved and discussed in [15-18].

The vibration of nanobeam is the most important and essential of the micro/nanobeam resonators. Alghamdi [9] studied the thermal damping of vibration of beam resonator with voids by dual-phase-lag generalized thermoelasticity theory. Youssef and Elsibai solved a problem of gold nanobeam by using state-space approach [19]. Youssef solved a problem of gold nanobeam with variable thermal conductivity by using the state-space approach [20]. Sharma and Grover discussed the thermal transfer and vibrations of an isotropic homogenous and thermoelastic micro/nanoscale thin beam resonators with voids [21]. Sun and Saka discussed the thermal damping vibration for microplate out-of-plane circular plate resonators [22]. They added a new factor based on Poisson’s ratio in the formula of thermoelastic damping which is different from the Lifshitz and Roukes formula [23]. Some authors discussed the vibration and the heat transfer process of thermoelastic nanobeams [24-28]. Eman and Youssef studied the vibration of gold nanobeam due to thermal shock [25]. Kiawa studied the effects of internal and external damping on transverse vibrations of a nanobeam due to a moving heat source by using the Green function properties [27]. Boley discussed the vibrations of a simply supported rectangular nanobeam subjected to a thermal shock distributed through its span [26]. A discussion of the thermally induced vibration of nano-beams structures has been done by Manolis and Beskos; they used a numerical method of analysis to the thermal of the elastic dynamic response of beam structure to thermal loading [28]. Al-Huniti et al. introduced an investigation of the thermally induced displacements and stresses of a heated rod by a high-power moving laser beam, and he studied the dynamical beheviour of the heated rod using the Laplace transforms technique [24].

Recently, the study of the viscoelastic materials and its relaxation times effects has become essential and vital in thermomechanics. The viscothermoelasticity theory and its variational principles in thermodynamics has been studied by Biot [29,30]. Drozdov [31] derived the constitutive model for the viscothermoelasticity behavior of polymer materials at finite strain. At the same time, Ezzat and El-Karmany [32] applied a new model of viscothermoelasticity for isotropic media to study the lag times impact of volume properties of viscothermoelasticity materials. Carcione et al. applied a new algorithm for wave simulations in an elastic medium by using the Kelvin–Voigt mechanical model [33]. Grover studied transverse vibrations in micro-scale viscothermoelastic beam resonators [34-36]. Sharma and Grover discussed the closed-form definition for the transverse vibrations of a homogenous thermoelastic fine beam with voids in micro/nanoscale [21]. Grover and Seth [37] studied viscothermoelastic micro-beam resonators based on the dual-phase-lag model.

2. Basic Equations

We assume an isotropic homogenous thermally conducting, Kelvin–Voigt type viscothermoelastic solid material in the Cartesian coordinate system. Initially, it is considered unstained and at the reference temperature T_0 everywhere. The essential governing partial differential equations of the motion and heat conduction have been assumed in the context of generalized thermoelasticity.

The displacement components $U(x,y,z;t)=(u,v,w)$ and absolute temperature $T(x,y,z;t)$, without body forces and heat sources, are given by [36]:

$$\sigma_{ij,j}=\rho\ddot{u}_i \tag{1}$$

[*] **Corresponding Author**
Mathematics Department, Al-Lith University College, Umm Al-Qura University, Al-Lith, Saudi Arabia.
E-mail address: ealehaibi@uqu.edu.sa

$$\sigma_{ij} = \lambda\delta_{ij}e_{kk} + 2\mu e_{ij} - \beta\delta_{ij}\left(T - T_0\right) \tag{2}$$

$$KT_{,ii} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\left(\rho C_\upsilon T + \beta T_0 \delta_{ij} e_{ij}\right) \tag{3}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{4}$$

where $i, j = x, y, z$, ρ is the density, α_T is the coefficient of linear thermal expansion, λ, μ Lamè's parameter, K is the thermal conductivity, τ_0 is the thermal relaxation time, and C_υ is the specific heat at constant strain.

For viscothermoelastic materials, Lamè's parameters have been considered in the form:

$$\lambda = \lambda_0\left(1 + \lambda_1 \frac{\partial}{\partial t}\right), \ \mu = \mu_0\left(1 + \mu_1 \frac{\partial}{\partial t}\right), \ \beta = \left(3\lambda + 2\mu\right)\alpha_T \tag{5}$$

where λ_1, μ_1 are the viscoelastic relaxation times and λ_0, μ_0 Lamè's parameter in the usual case.

3. Formulation of the problem

A small flexural deflection of a thin viscothermoelastic nanobeam of length ℓ , width b , and thickness h has been considered. The x, y, and z-axes are defined along the longitudinal $\left(0 \le x \le \ell\right)$, width $\left(-b/2 \le y \le b/2\right)$, thickness $\left(-h/2 \le z \le h/2\right)$, and directions of the beam, respectively.

In a state of equilibrium, the beam is undeformed, unstressed, without a damping mechanism, and the temperature is T_0 everywhere [6].

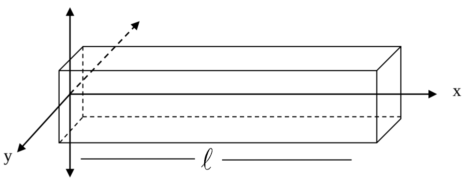

Figure 1: The rectangular beam in the Cartesian coordinate system

The usual Euler–Bernoulli assumption [34] has been considered, then, the cross-section of any plane, initially perpendicular to the x-axis remains plane and perpendicular to the x-axis during bending. Therefore, the displacements are given by:

$$u = -z\frac{\partial w\left(x,t\right)}{\partial x} \ , \ v = 0 \ , \ w\left(x,y,z,t\right) = w\left(x,t\right) \tag{6}$$

Where $w\left(x,t\right)$ is the lateral deflection.

The flexural moment of the cross-section is given by

$$M(x,t) + \rho A\frac{\partial^2 w\left(x,t\right)}{\partial t^2} = 0 \tag{7}$$

where $M_T(x,t)$ is defined as the thermal moment of the beam and is given by:

$$M_T(x,t) = b\int_{-h/2}^{h/2} \theta(x,z,t)\, z\, dz \tag{8}$$

and $M(x,t)$ is given by:

$$M\left(x,t\right) = -b\int_{-h/2}^{h/2}\sigma_{xx}\left(x,t,z\right)z\,dz = \left(\lambda + 2\mu\right) I\frac{\partial^2 w(x,t)}{\partial x^2} + \beta M_T(x,t) \tag{9}$$

Where I is the moment of inertia of the cross-section about the x-axis and is given by $I = \left(bh^3/12\right)$ Hence, the differential equation of thermally induced lateral vibration of the beam may be expressed in the form [34]:

$$\left(\lambda + 2\mu\right)I\frac{\partial^4 w\left(x,t\right)}{\partial x^4} + \rho A\frac{\partial^2 w\left(x,t\right)}{\partial t^2} + \beta\frac{\partial^2 M_T\left(x,t\right)}{\partial x^2} = 0 \tag{10}$$

where the area of the beam cross-section is $A = \left(hb\right)$ and $\theta\left(x,z,t\right) = \left(T\left(x,z,t\right) - T_0\right)$ is the temperature increment of the beam.

The non-Fourier heat conduction equation has the following form [34]:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\theta\left(x,z,t\right) = \left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)\left(\frac{\rho C_\upsilon}{k}\theta\left(x,z,t\right) + \frac{\left(3\lambda + 2\mu\right)\alpha_T T_0}{k}e\left(x,z,t\right)\right) \tag{11}$$

where $e\left(x,z,t\right)$ is the volumetric strain which is given by:

$$e\left(x,z,t\right) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \tag{12}$$

By using the forms in [6], we obtain that:

$$e\left(x,z,t\right) = -z\frac{\partial^2 w\left(x,t\right)}{\partial x^2} \tag{13}$$

From the relations in [5], we have the following:

$$\lambda + 2\mu = \left(\lambda_0 + 2\mu_0\right)\left(1 + \beta_1\frac{\partial}{\partial t}\right), \ \left(3\lambda + 2\mu\right) = \left(3\lambda_0 + 2\mu_0\right)\left(1 + \beta_2\frac{\partial}{\partial t}\right) \tag{14}$$

where $\beta_1 = \frac{\left(\lambda_0\lambda_1 + 2\mu_0\mu_1\right)}{\left(\lambda_0 + 2\mu_0\right)}$, $\beta_2 = \frac{\left(3\lambda_0\lambda_1 + 2\mu_0\mu_1\right)}{\left(3\lambda_0 + 2\mu_0\right)}$ are called the aggregation of the viscoelastic relaxation times parameters.

Because there is no heat source or heat flow across the upper and lower surfaces of the beam, $\frac{\partial}{\partial z}\theta\left(x, \pm\frac{h}{2}, t\right) = 0$. Hence, For a very long and skinny beam, the temperature increment varies in terms of a $\sin\left(pz\right)$ function along the thickness direction, gives [38]:

$$\theta\left(x,z,t\right) = \vartheta\left(x,t\right)\sin\left(pz\right) \tag{15}$$

where $p = \pi/h$.

Hence, equations [8], [10], and [15] gives:

$$\left(\lambda_0 + 2\mu_0\right)\left(1 + \beta_1\frac{\partial}{\partial t}\right)\frac{\partial^4 w\left(x,t\right)}{\partial x^4} + \frac{12\rho}{h^2}\frac{\partial^2 w\left(x,t\right)}{\partial t^2} + \frac{12\left(3\lambda_0 + 2\mu_0\right)\alpha_T}{h^3}\left(1 + \beta_2\frac{\partial}{\partial t}\right)\frac{\partial^2\vartheta\left(x,t\right)}{\partial x^2}\int_{-h/2}^{h/2} z\sin\left(pz\right)dz = 0 \tag{16}$$

and equation [11] gives

$$\left(\frac{\partial^2}{\partial x^2} - p^2\right)\vartheta\left(x,t\right)\sin\left(pz\right) = \left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)\left(\frac{\rho C_\upsilon}{k}\vartheta\left(x,t\right)\sin\left(pz\right) - \frac{\left(3\lambda_0 + 2\mu_0\right)\alpha_T T_0}{k}\left(1 + \beta_2\frac{\partial}{\partial t}\right)z\frac{\partial^2 w\left(x,t\right)}{\partial x^2}\right) \tag{17}$$

By doing the integrations, the equation [16] takes the form

$$\left(\lambda_0 + 2\mu_0\right)\left(1 + \beta_1\frac{\partial}{\partial t}\right)\frac{\partial^4 w\left(x,t\right)}{\partial x^4} + \frac{12\rho}{h^2}\frac{\partial^2 w\left(x,t\right)}{\partial t^2} + \frac{24\left(3\lambda_0 + 2\mu_0\right)\alpha_T}{h\pi^2}\left(1 + \beta_2\frac{\partial}{\partial t}\right)\frac{\partial^2\vartheta\left(x,t\right)}{\partial x^2} = 0 \tag{18}$$

In the equation [17], we multiply both sides by "z" and integrating concerning "z" from $-h/2$ to $h/2$, then we obtain

$$\left(\frac{\partial^2}{\partial x^2} - p^2\right)\vartheta\left(x,t\right) = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\left(\varepsilon\,\vartheta\left(x,t\right) - \frac{T_0 h\pi^2\left(3\lambda_0 + 2\mu_0\right)\alpha_T}{24k}\left(1 + \beta_2\frac{\partial}{\partial t}\right)\frac{\partial^2 w\left(x,t\right)}{\partial x^2}\right) \tag{19}$$

Where $\varepsilon = \frac{\rho C_\upsilon}{k}$.

For simplicity, we will use the following dimensionless variables [19]:

$$\left(x', w', h', \ell'\right) = \varepsilon c_0\left(x, w, h, \ell\right), \left(t', \tau_0', \beta_1', \beta_2'\right) = \varepsilon c_0^2\left(t, \tau_0, \beta_1, \beta_2\right), \sigma' = \frac{\sigma}{\lambda_0 + 2\mu_0}, \vartheta' = \frac{\vartheta}{T_0} \tag{20}$$

where $c_0^2 = \frac{\lambda_0 + 2\mu_0}{\rho}$.

Thus, we get

$$\left(1 + \beta_1\frac{\partial}{\partial t}\right)\frac{\partial^4 w\left(x,t\right)}{\partial x^4} + \varepsilon_1\frac{\partial^2 w\left(x,t\right)}{\partial t^2} = -\varepsilon_2\left(1 + \beta_2\frac{\partial}{\partial t}\right)\frac{\partial^2\vartheta\left(x,t\right)}{\partial x^2} \tag{21}$$

and

$$\left[\left(\frac{\partial^2}{\partial x^2}-\varepsilon_3\right)-\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)\right]\vartheta(x,t)=-\varepsilon_4\left(1+\beta_2\frac{\partial}{\partial t}\right)\frac{\partial^2 w(x,t)}{\partial x^2} \tag{22}$$

$$\sigma_{xx}(x,z,t)=\left(1+\beta_1\frac{\partial}{\partial t}\right)e(x,,z,t)-\gamma\left(1+\beta_2\frac{\partial}{\partial t}\right)\vartheta(x,t)\sin(pz) \tag{23}$$

where

$$\varepsilon_1=\frac{12}{h^2},\ \varepsilon_2=\frac{24\gamma}{\pi^2 h},\ \varepsilon_3=p^2,\ \varepsilon_4=\frac{\pi^2 h(3\lambda_0+2\mu_0)\alpha_T}{24k\varepsilon},\ \gamma=\frac{(3\lambda_0+2\mu_0)\alpha_T T_0}{(\lambda_0+2\mu_0)}.$$

(The primes have been dropped for convenience)

3.1. The formulation in the Laplace transform domain

We will apply the Laplace transform for equations [21] and [22], which is defined by:

$$\bar{f}(s)=\int_0^\infty f(t)e^{-st}\,dt \tag{24}$$

Hence, we obtain the following system of ordinary differential equations:

$$(1+\beta_1 s)\frac{d^4\bar{w}}{dx^4}+\varepsilon_1 s^2\bar{w}(x,s)=-\varepsilon_2(1+\beta_2 s)\frac{d^2\bar{\vartheta}}{dx^2} \tag{25}$$

$$\left(\frac{\partial^2}{\partial x^2}-\varepsilon_3-\left(s+\tau_o s^2\right)\right)\bar{\vartheta}=-\varepsilon_4\left(s+\tau_o s^2\right)(1+\beta_2 s)\frac{d^2\bar{w}}{dx^2} \tag{26}$$

$$\bar{\sigma}_{xx}=(1+\beta_1 s)\bar{e}-\gamma(1+\beta_2 s)\bar{\vartheta}\sin(pz) \tag{27}$$

$$\bar{e}=-z\frac{d^2\bar{w}}{dx^2} \tag{28}$$

Within applying the Laplace transform, we used the following initial conditions:

$$\vartheta(x,0)=w(x,0)=\frac{\partial\vartheta(x,0)}{\partial t}=\frac{\partial w(x,0)}{\partial t}=0 \tag{29}$$

We can re-write the above system to be in the forms:

$$\left(D^4+\varepsilon_5\right)\bar{w}=-\varepsilon_6 D^2\bar{\vartheta} \tag{30}$$

and

$$\left(D^2-\varepsilon_7\right)\bar{\vartheta}=-\varepsilon_8 D^2\bar{w} \tag{31}$$

where $D^r=\frac{d^r}{dx^r}$, $\varepsilon_5=\frac{\varepsilon_1 s^2}{(1+\beta_1 s)}$, $\varepsilon_6=\frac{\varepsilon_2(1+\beta_2 s)}{(1+\beta_1 s)}$,

$\varepsilon_7=\varepsilon_3+\left(s+\tau_o s^2\right),\varepsilon_8=\varepsilon_4\left(s+\tau_o s^2\right)(1+\beta_2 s)$.

Eliminating $\bar{w}$ between the equations of the above system, then, we get

$$\left[D^6-LD^4+MD^2-N\right]\bar{\vartheta}=0 \tag{32}$$

Similarly, eliminating $\bar{\vartheta}$ gives:

$$\left[D^6-LD^4+MD^2-N\right]\bar{w}=0 \tag{33}$$

where $L=\varepsilon_7+\varepsilon_6\varepsilon_8,\ M=\varepsilon_5,\ N=\varepsilon_5\varepsilon_7$.

The solutions of the equations [32] and [33] take the forms:

$$\bar{\vartheta}(x,s)=-\varepsilon_8\sum_{i=1}^{3}c_i k_i^2\sinh\left(k_i(\ell-x)\right) \tag{34}$$

and

$$\bar{w}(x,s)=\sum_{i=1}^{3}c_i\left(k_i^2-\varepsilon_7\right)\sinh\left(k_i(\ell-x)\right) \tag{35}$$

Where the parameters $\pm k_1,\pm k_2,\pm k_3$ are the roots of the following characteristic equation:

$$k^6-Lk^4+Mk^2-N=0 \tag{36}$$

To calculate the constants $c_i=c_i(s),\ i=1,2,3$, we must apply any set of boundary conditions, so we consider that the beam is thermally loaded and simply supported as following:

$$w(0,t)=\left.\frac{\partial^2 w(x,t)}{\partial x^2}\right|_{x=0}=0,\ \vartheta(0,t)=\vartheta_0 g(t) \tag{37}$$

and

$$w(\ell,t)=\left.\frac{\partial^2 w(x,t)}{\partial x^2}\right|_{x=\ell}=\vartheta(\ell,t)=0 \tag{38}$$

Where ϑ_0 is constant.

Apply the Laplace transform, we have:

$$\bar{w}(0,s)=\frac{\partial^2\bar{w}(0,s)}{\partial x^2}=0,\ \bar{\vartheta}(0,s)=\vartheta_0\bar{g}(s) \tag{39}$$

and

$$\bar{w}(\ell,s)=\frac{\partial^2\bar{w}(\ell,s)}{\partial x^2}=\bar{\vartheta}(\ell,s)=0 \tag{40}$$

Then, we obtain the following system of linear equations:

$$\sum_{i=1}^{3}c_i k_i^2\sinh(k_i\ell)=-\frac{\vartheta_0\bar{g}(s)}{\varepsilon_8} \tag{41}$$

$$\sum_{i=1}^{3}c_i\left(k_i^2-\varepsilon_7\right)\sinh(k_i\ell)=0 \tag{42}$$

and

$$\sum_{i=1}^{3}c_i\left(k_i^2-\varepsilon_7\right)k_i^2\sinh(k_i\ell)=0 \tag{43}$$

After solving the above system, then, we get the solutions in the Laplace transform domain as follows:

$$\theta(x,s)=\frac{\vartheta_0\bar{g}(s)\sin(pz)}{\varepsilon_7}\left[\begin{array}{l}\frac{\left(\varepsilon_7-k_2^2\right)\left(\varepsilon_7-k_3^2\right)k_1^2}{\left(k_1^2-k_2^2\right)\left(k_1^2-k_3^2\right)\sinh(k_1\ell)}\sinh\left(k_1(\ell-x)\right)+\\ \frac{\left(\varepsilon_7-k_1^2\right)\left(\varepsilon_7-k_3^2\right)k_2^2}{\left(k_2^2-k_1^2\right)\left(k_2^2-k_3^2\right)\sinh(k_2\ell)}\sinh\left(k_2(\ell-x)\right)+\\ \frac{\left(\varepsilon_7-k_1^2\right)\left(\varepsilon_7-k_2^2\right)k_3^2}{\left(k_3^2-k_1^2\right)\left(k_3^2-k_2^2\right)\sinh(k_3\ell)}\sinh\left(k_3(\ell-x)\right)\end{array}\right] \tag{44}$$

$$\bar{w}(x,s)=-\frac{\vartheta_0\bar{g}(s)\left(\varepsilon_7-k_1^2\right)\left(\varepsilon_7-k_2^2\right)\left(\varepsilon_7-k_3^2\right)}{\varepsilon_7\varepsilon_8}\left[\begin{array}{l}\frac{1}{\left(k_1^2-k_2^2\right)\left(k_1^2-k_3^2\right)\sinh(k_1\ell)}\sinh\left(k_1(\ell-x)\right)+\\ \frac{1}{\left(k_2^2-k_1^2\right)\left(k_2^2-k_3^2\right)\sinh(k_2\ell)}\sinh\left(k_2(\ell-x)\right)+\\ \frac{1}{\left(k_3^2-k_1^2\right)\left(k_3^2-k_2^2\right)\sinh(k_3\ell)}\sinh\left(k_3(\ell-x)\right)\end{array}\right] \tag{45}$$

and

$$\bar{e}(x,s)=\frac{z\vartheta_0\bar{g}(s)\left(\varepsilon_7-k_1^2\right)\left(\varepsilon_7-k_2^2\right)\left(\varepsilon_7-k_3^2\right)}{\varepsilon_7\varepsilon_8}\left[\begin{array}{l}\frac{k_1^2}{\left(k_1^2-k_2^2\right)\left(k_1^2-k_3^2\right)\sinh(k_1\ell)}\sinh\left(k_1(\ell-x)\right)+\\ \frac{k_2^2}{\left(k_2^2-k_1^2\right)\left(k_2^2-k_3^2\right)\sinh(k_2\ell)}\sinh\left(k_2(\ell-x)\right)+\\ \frac{k_3^2}{\left(k_3^2-k_1^2\right)\left(k_3^2-k_2^2\right)\sinh(k_3\ell)}\sinh\left(k_3(\ell-x)\right)\end{array}\right] \tag{46}$$

3.2. The Stress and the Strain-Energy

The stress-strain energy through the beam is calculated by [39]:

$$\varpi(x,z,t)=\sum_{i,j=1}^{3}\frac{1}{2}\sigma_{ij}e_{ij}=\frac{1}{2}\sigma_{xx}(x,z,t)e(x,z,t) \tag{47}$$

Hence, we have:

$$\varpi(x,z,t)=\frac{1}{2}\left[L^{-1}\left(\bar{\sigma}_{xx}(x,z,s)\right)\right]\left[L^{-1}\left(\bar{e}(x,z,s)\right)\right] \tag{48}$$

where $L^{-1}[\bullet]$ gives the inversion of Laplace transform.

4. Numerical Inversion of the Laplace Transform

Before we go on the numerical solutions of the problem, we must determine the function of the thermal loading $g(t)$. We consider that the thermal loading will take two different types as follows:

1- Ramp-type heating

$$g(t)=\begin{cases} 0 & t<0 \\ \dfrac{t}{t_0} & 0<t<t_0 \\ 1 & t\geq t_0 \end{cases} \tag{49}$$

Where t_0 is called the ramping time parameter

Applying Laplace transform defined in [24], then, we have

$$\bar{g}(s)=\frac{1-e^{-st_0}}{s^2 t_0} \tag{50}$$

2- Harmonic-type heating

$$g(t)=\sin(\omega t) \tag{51}$$

where ω is called the angular thermal moment parameter

By using Laplace transform defined in [24], we get

$$\bar{g}(s)=\frac{\omega}{s^2+\omega^2} \tag{51}$$

The approximation method of Riemann-sum has been used to obtain numerical results. Within this method, any function in the Laplace domain can be inverted to the time-domain as:

$$f(t)=\frac{e^{\kappa t}}{t}\left[\frac{1}{2}\bar{f}(\kappa)+\mathrm{Re}\sum_{n=1}^{N}(-1)^n\,\bar{f}\left(\kappa+\frac{i\,n\pi}{t}\right)\right] \tag{52}$$

where Re is the real part and i is an imaginary number unit. For faster convergence, numerous numerical experiments have shown that the value of κ satisfies the relation $\kappa t\approx 4.7$ Tzou [2].

5. Numerical Results and Discussion

Now, we will consider a numerical example for which computational results are given. For this purpose, The gold (Au) has been taken as the thermoelastic material for which we take the following values of the different physical constants[39]:

$k=318\ W/(mK)$, $\alpha_T=14.2(10)^{-6}\ K^{-1}$,

$\rho=1930\ kg/m^3$, $T_0=293K$,

$C_\upsilon=130\ J/(kg\,K)$, $\lambda_0=458.33\times10^9\ N/m^2$,

$\mu_0=62.5\times10^9\ N/m^2$, $\tau_0=4.32\times10^{-13}\ s$,

$\lambda_1=\mu_1=6.89\times10^{-13}\ s$.

The aspect ratios of the beam are fixed as $\ell/h=5$ and $b=h/2$. For the nanoscale beam, we will take the range of the beam length ℓ $(1-100)\times10^{-12}\,m$ —the original time t and the relaxation time τ_0 of order 10^{-12} sec and 10^{-14} sec, respectively.

The figures were prepared by using the non-dimensional variables for beam length $\ell=1.0$, $\theta_0=1.0$ $z=h/4$ and $t=1.0$.

Figures 2-6 represent the temperature increment, the lateral vibration, the deformation, the stress, and the stress-strain energy distributions, respectively, for the thermoelastic case and the viscothermoelastic case. It has been noted that the temperature increment almost is the same for the two cases, while the effects of the viscothermoelastic parameters are significant on the lateral vibration, the deformation, the stress, and the stress-strain energy distributions. The peak points of the lateral vibration, deformation, stress, and stress-strain energy distributions are raised in the case of viscothermoelasticity.

Figures 7-11 represent the temperature increment, the lateral vibration, the deformation, the stress, and the stress-strain energy distributions, respectively, with variance values of ramping time parameter $t_0=(0.5,1.0,1.5)$ and the viscothermoelastic case. It has been noted that the temperature increment in the two cases $t_0=0.5$ and $t_0=1.0$ are almost the same, while it is different in the case $t_0=1.5$. The values of the temperature increment, the lateral vibration, the deformation, the stress, and the stress-strain energy decrease when the value of the ramping time parameter increases. The values of the beak points of the lateral vibration, the deformation, the stress, and the stress-strain energy increase when the value of the ramping time parameter decreases.

Figures 12-16 represent the temperature increment, the lateral vibration, the deformation, the stress, and the stress-strain energy distributions, respectively, for variance values of the angular thermal moment parameter $\omega=(\pi/6,\pi/3,\pi/2)$ to stand on the effect of this parameter on all the studied functions. It has been noted that the effect of the angular thermal moment parameter is significant in all the studied functions. When the value of the angular thermal moment parameter increases, the values of the temperature increment, the lateral vibration, the deformation, the stress, and the stress-strain energy distributions increase.

6. Conclusion

In this work, a simply supported viscothermoelastic nanobeam has been thermally loaded by ram-type and harmonic-type heating. The viscothermoelastic parameters, ramping time parameter, and angular thermal moment parameter have significant effects on the temperature increment, the lateral vibration, the deformation, the stress, and the stress-strain energy distributions.

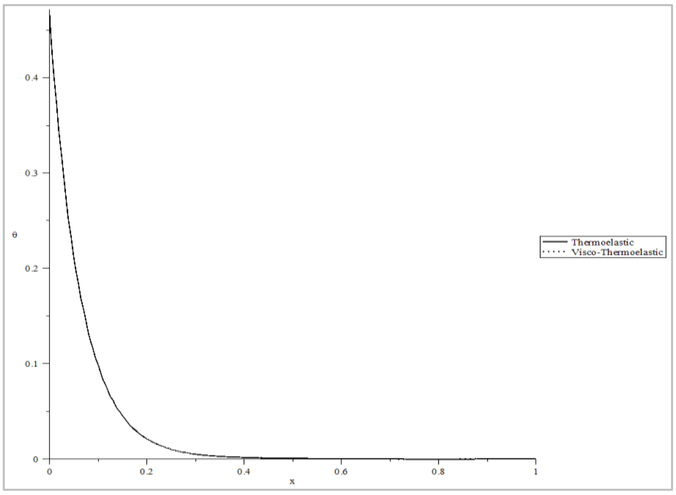

Figure 2: The temperature increment distribution for different models

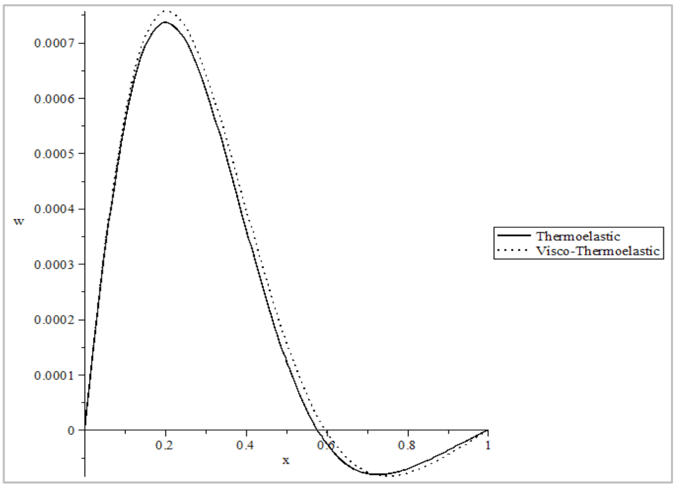

Figure 3: The lateral vibration distribution for different models

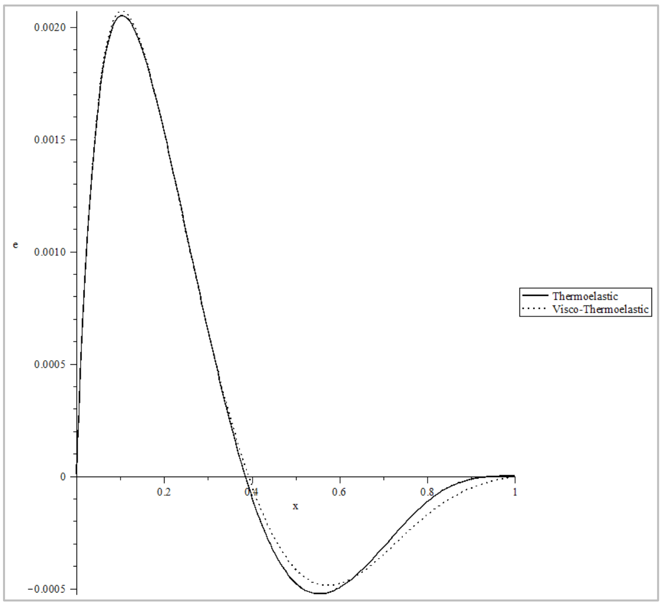

Figure 4: The deformation distribution for different models

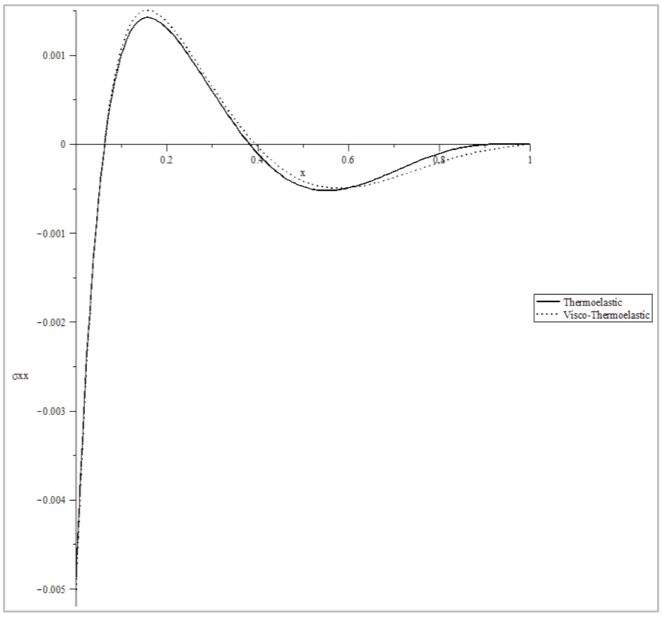

Figure 5: The stress distribution for different models

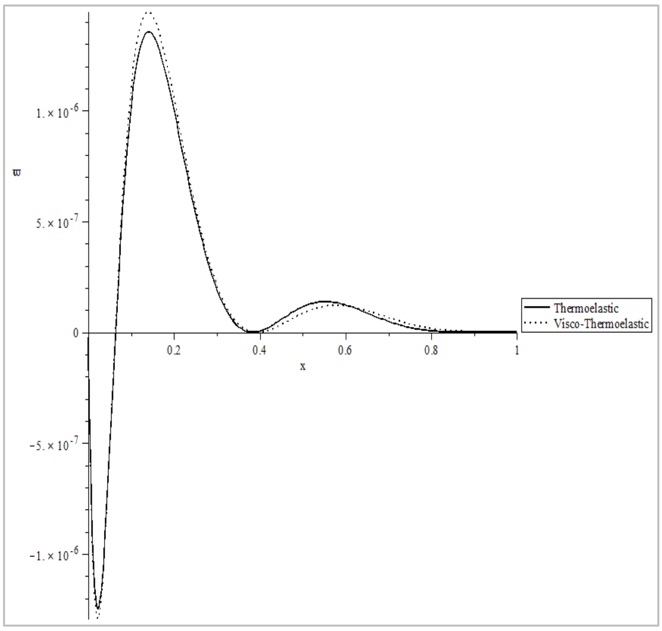

Figure 6: The strain-energy distribution for different models

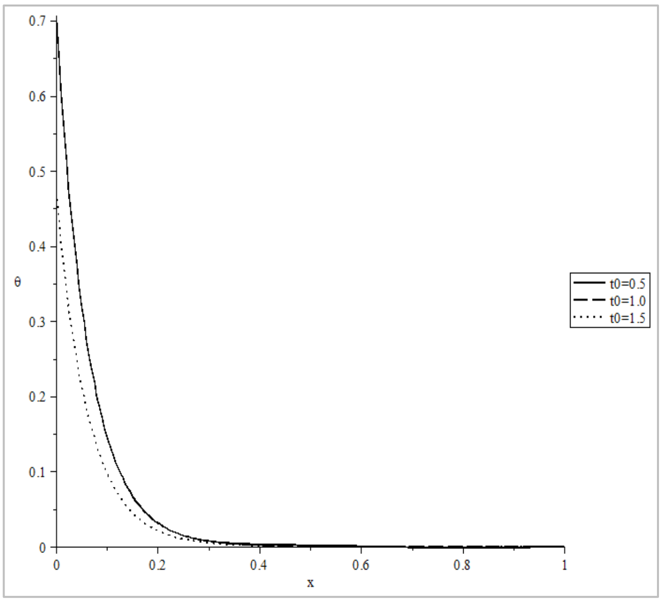

Figure 7: The temperature increments distribution with variance values of ramping time parameter

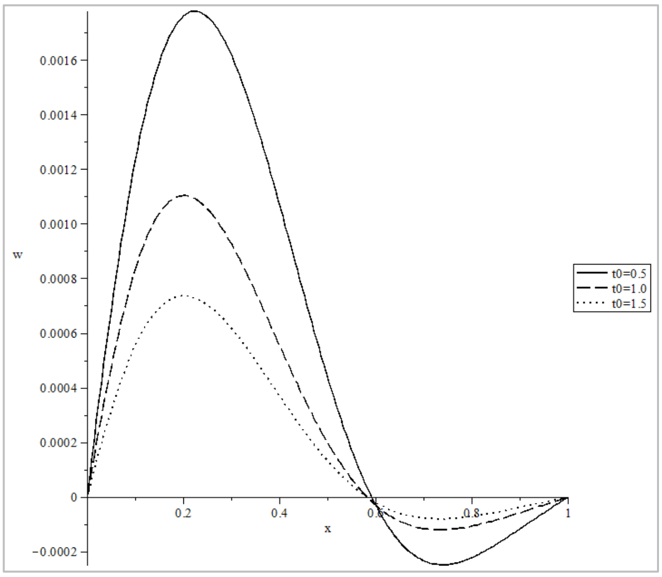

Figure 8: The lateral deflection distribution with variance values of ramping time parameter

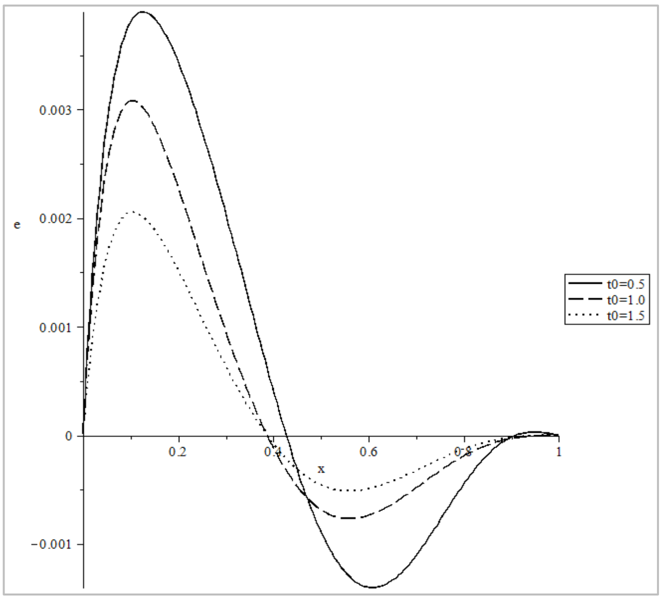

Figure 9: The deformation distribution with variance values of ramping time parameter

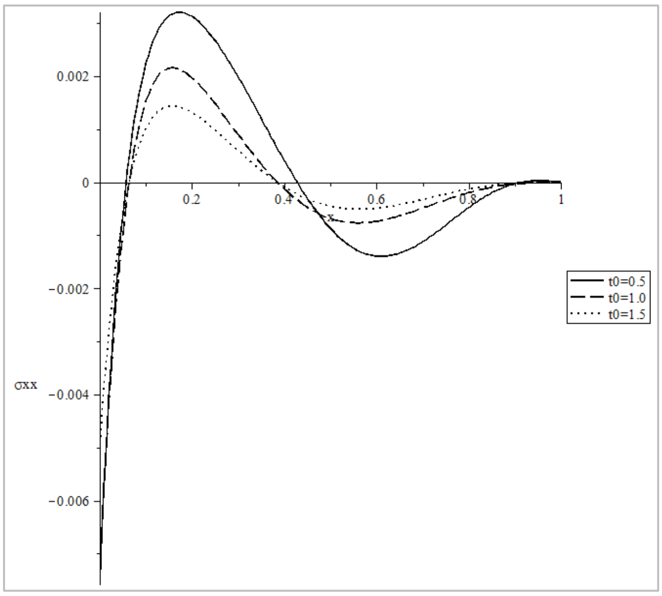

Figure 10: The stress distribution with variance values of ramping time parameter

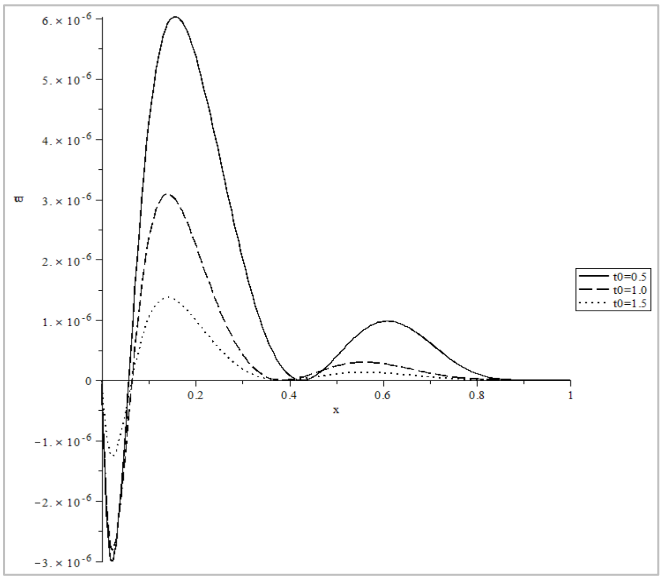

Figure 11: The stress-strain energy distribution with variance values of ramping time parameter

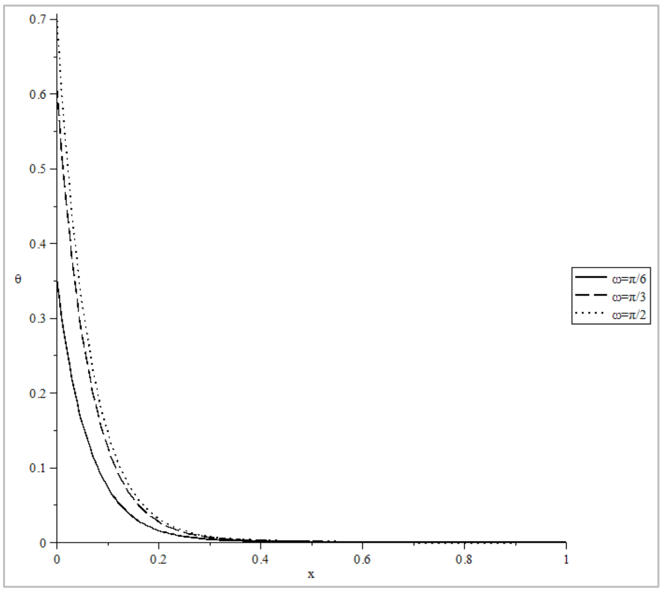

Figure 12: The temperature increments distribution for variance values of angular thermal moment parameter

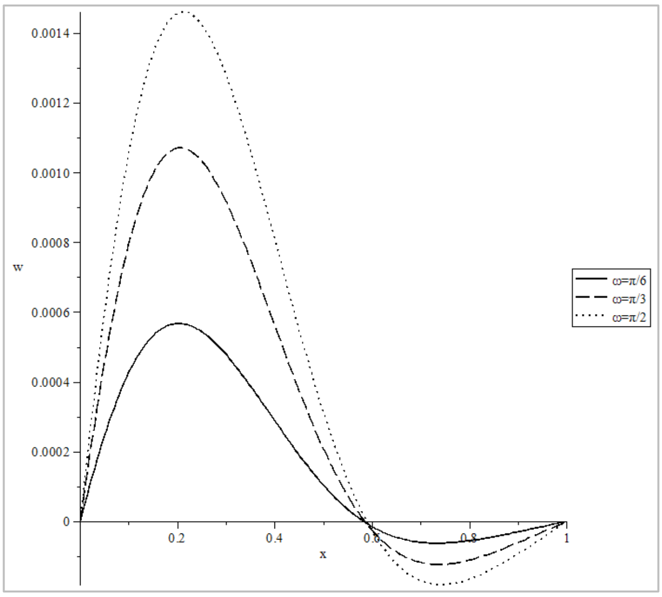

Figure 13: The lateral deflection distribution for variance values of angular thermal moment parameter

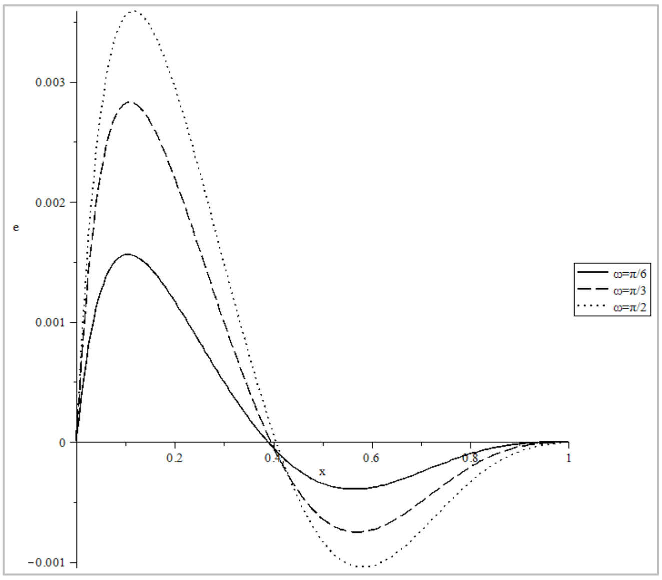

Figure 14: The deformation distribution for variance values of angular thermal moment parameter

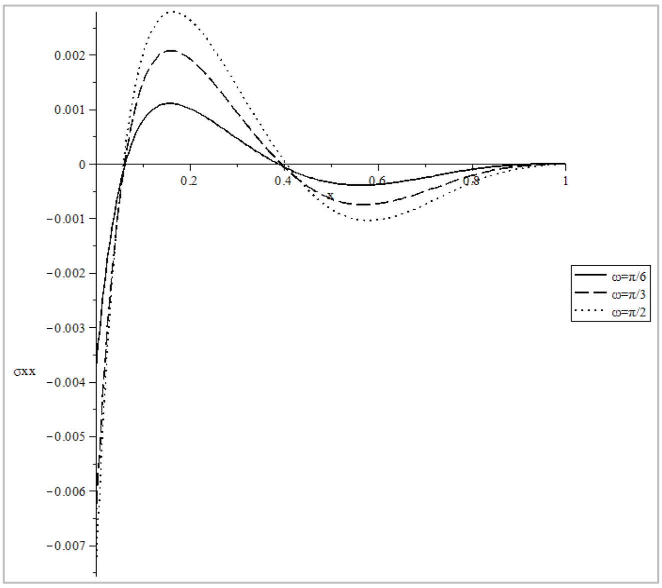

Figure 15: The stress distribution for variance values of angular thermal moment parameter

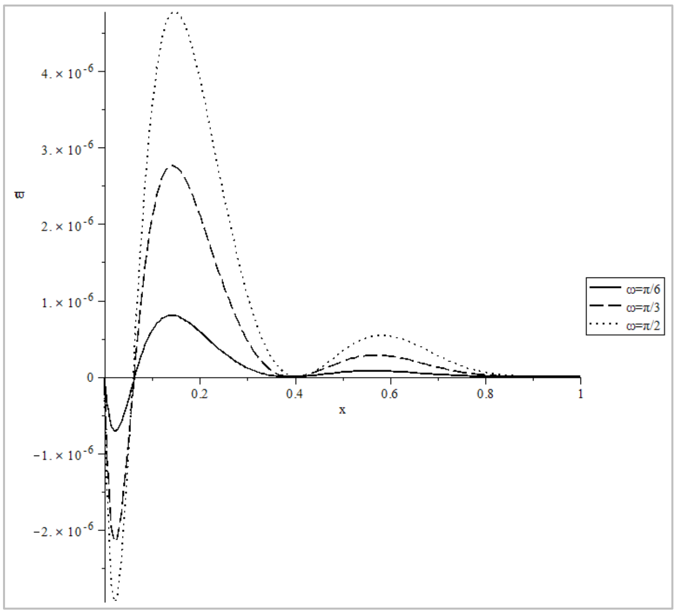

Figure 16: The stress-strain energy distribution for variance values of angular thermal moment parameter

References

[1] D. Tzou, On the thermal shock wave induced by a moving heat source. *J. Heat Transf.*, **1989**, *111*, 232-238. https://doi.org/10.1115/1.3250667

[2] D. Tzou, M.-t.M.H. Transfer, The Lagging Behavior, Taylor Francis, Washington (**1997**).

[3] M. Xu, J. Guo, L. Wang, L. Cheng, Thermal wave interference as the origin of the overshooting phenomenon in dual-phase-lagging heat conduction, *Int. J. Therm. Sci.*, **2011**, *50*, 825-830. https://doi.org/10.1016/j.ijthermalsci.2010.12.006

[4] N.S. Al-Huniti, M. Al-Nimr, Thermoelastic behavior of a composite slab under a rapid dual-phase-lag heating. *J. Therm. Stress.*, **2004**, *27*, 607-623. https://doi.org/10.1080/01495730490466200

[5] J.-R. Ho, C.-P. Kuo, W.-S. Jiaung, Study of heat transfer in multilayered structure within the framework of dual-phase-lag heat conduction model using lattice Boltzmann method. *Int. J. Heat Mass Transf.*, **2003**, *46*, 55-69. https://doi.org/10.1016/S0017-9310(02)00260-0

[6] Y.-M. Lee, T.-W. Tsai, Ultra-fast pulse-laser heating on a two-layered semi-infinite material with interfacial contact conductance. *Int. Commun. Heat Mass Transf.*, **2007**, *34*, 45-51. https://doi.org/10.1016/j.icheatmasstransfer.2006.08.017

[7] K.-C. Liu, Numerical analysis of dual-phase-lag heat transfer in a layered cylinder with nonlinear interface boundary conditions. *Comput. Phys. Commun.*, **2007**, *177*, 307-314. https://doi.org/10.1016/j.cpc.2007.02.110

[8] K. Ramadan, Semi-analytical solutions for the dual phase lag heat conduction in multilayered media. *Int. J. Therm. Sci.*, **2009**, *48*, 14-25. https://doi.org/10.1016/j.ijthermalsci.2008.03.004

[9] N. Alghamdi, Dual-Phase-Lagging Thermoelastic Damping Vibration in Micro-Nano Scale Beam Resonators with Voids. *Int. J. Multidisciplinary and Current Research* 5 (**2017**).

[10] N.A. Alghamdi, H.M. Youssef, Dual-phase-lagging thermoelastic damping in-extensional vibration of rotating nano-ring. *Microsyst. Technol.*, **2017**, *23*, 4333-4343. https://doi.org/10.1007/s00542-017-3294-z

[11] M.A. Biot, Thermoelasticity and irreversible thermodynamics. *J. Appl. Phys.*, **1956**, *27*, 240-253. https://doi.org/10.1063/1.1722351

[12] H. Youssef, N. Alghamdi, Thermoelastic damping in nanomechanical resonators based on two-temperature generalized thermoelasticity theory. *J. Therm. Stresses*, **2015**, *38*, 1345-1359. https://doi.org/10.1080/01495739.2015.1073541

[13] H.W. Lord, Y. Shulman, A generalized dynamical theory of thermoelasticity. *J. Mech. Phys. Solids*, **1967**, *15*, 299-309. https://doi.org/10.1016/0022-5096(67)90024-5

[14] R.S. Dhaliwal, H.H. Sherief, Generalized thermoelasticity for anisotropic media. *Q. Appl. Math.*, **1980**, *38*, 1-8.

[15] C.M. Hoang, Thermoelastic damping depending on vibration modes of nano beam resonator. *Commun. Phys.*, **2015**, *25*, 317. DOI: 10.15625/0868-3166/25/4/6887

[16] A.K. Naik, M. Hanay, W. Hiebert, X. Feng, M.L. Roukes, Towards single-molecule nanomechanical mass spectrometry. *Nat. Nanotechnol.*, **2009**, *4*, 445-450. https://doi.org/10.1038/nnano.2009.152

[17] A.D. O'Connell, M. Hofheinz, M. Ansmann, R.C. Bialczak, M. Lenander, E. Lucero, M. Neeley, D. Sank, H. Wang, M. Weides, Quantum ground state and single-phonon control of a mechanical resonator. *Nature*, **2010**, *464*, 697-703. https://doi.org/10.1038/nature08967

[18] J. Van Beek, R. Puers, A review of MEMS oscillators for frequency reference and timing applications. *J. Micromech. Microeng.*, **2011**, *22*, 013001.

[19] H.Youssef, K.A. Elsibai, Vibration of gold nanobeam induced by different types of thermal loading—a state-space approach. *Nanoscale Microscale Thermophys. Eng.*, **2011**, *15*, 48-69. https://doi.org/10.1080/15567265.2010.549929

[20] H. Youssef, Vibration of gold nanobeam with variable thermal conductivity: state-space approach. *Appl. Nanosci.*, **2013**, *3*, 397-407. DOI 10.1007/s13204-012-0158-9

[21] J. Sharma, D. Grover, Thermoelastic vibrations in micro-/nano-scale beam resonators with voids. *J. Sound Vib.*, **2011**, *330*, 2964-2977. https://doi.org/10.1016/j.jsv.2011.01.012

[22] Y. Sun, M. Saka, Thermoelastic damping in micro-scale circular plate resonators. *J. Sound and Vibration*, **2010**, *329*, 328-337. https://doi.org/10.1016/j.jsv.2009.09.014

[23] R. Lifshitz, M.L. Roukes, Thermoelastic damping in micro-and nanomechanical systems. *Physical Review B*, **2000**, *61*, 5600. https://doi.org/10.1103/PhysRevB.61.5600

[24] N.S. Al-Huniti, M. Al-Nimr, M. Naji, Dynamic response of a rod due to a moving heat source under the hyperbolic heat conduction model. *J. Sound Vib.*, **2001**, *242*, 629-640. https://doi.org/10.1006/jsvi.2000.3383

[25] E.A. Al-Lehaibi, H.M. Youssef, Vibration of gold nanobeam with variable young's modulus due to thermal shock. *World J. Nano Sci. Eng.*, **2015**, *5*, 194. DOI: 10.4236/wjnse.2015.54020

[26] B.A. Boley, Approximate analyses of thermally induced vibrations of beams and plates. *J. Appl. Mech.*, **1972**, *39*, 212-216. https://doi.org/10.1115/1.3422615

[27] J. Kidawa-Kukla, Application of the Green functions to the problem of the thermally induced vibration of a beam. *J. Sound Vibration*, **2003**, *262*, 865-876. https://doi.org/10.1016/S0022-460X(02)01133-1

[28] G. Manolis, D. Beskos, Thermally induced vibrations of beam structures, *Comput. Methods Appl. Mech. Eng.*, **1980**, *21*, 337-355. https://doi.org/10.1016/0045-7825(80)90101-2

[29] M.A. Biot, Theory of stress-strain relations in anisotropic viscoelasticity and relaxation phenomena. *J. Appl. Phys.*, **1954**, *25*, 1385-1391. DOI:10.1063/1.1721573

[30] M.A. Biot, Variational principles in irreversible thermodynamics with application to viscoelasticity. *Physical Review*, **1955**, *97*, 1463. https://doi.org/10.1103/PhysRev.97.1463

[31] A. Drozdov, A constitutive model in finite thermoviscoelasticity based on the concept of transient networks. *Acta Mechanica*, **1999**, *133*, 13-37. https://doi.org/10.1007/BF01179008

[32] M.A. Ezzat, A.S. El-Karamany, The relaxation effects of the volume properties of viscoelastic material in generalized thermoelasticity. *Int. J. Eng. Sci.*, **2003**, *41*, 2281-2298. https://doi.org/10.1016/S0020-7225(03)00108-3

[33] J.M. Carcione, F. Poletto, D. Gei, 3-D wave simulation in anelastic media using the Kelvin–Voigt constitutive equation. *J. Comput. Phys.*, **2004**, *196*, 282-297. https://doi.org/10.1016/j.jcp.2003.10.024

[34] D. Grover, Viscothermoelastic vibrations in micro-scale beam resonators with linearly varying thickness. *Can. J. Phys.*, **2012**, *90*, 487-496. https://doi.org/10.1139/p2012-044

[35] D. Grover, Damping in thin circular viscothermoelastic plate resonators. *Can. J. Phys.*, **2015**, *93*, 1597-1605. https://doi.org/10.1139/cjp-2014-0575

[36] D. Grover, Transverse vibrations in micro-scale viscothermoelastic beam resonators. *Arch. Appl. Mech.*, **2013**, *83*, 303-314. https://doi.org/10.1007/s00419-012-0656-y

[37] D. Grover, R. Seth, Viscothermoelastic micro-scale beam resonators based on dual-phase lagging model. *Microsyst. Technol.*, **2018**, *24*, 1667-1672. https://doi.org/10.1007/s00542-017-3515-5

[38] H. Youssef, N.A. Al-Ghamdi, Vibration of Gold Nano Beam in Context of Two-Temperature Generalized Thermoelasticity without energy dissipation, ICTEA: International Conference on Thermal Engineering, **2017**.

[39] K.A. Elsibai, H.M. Youssef, State-space approach to vibration of gold nanobeam induced by ramp type heating without energy dissipation in femtoseconds scale. *J. Therm. Stresses*, **2011**, *34*, 244-263. https://doi.org/10.1007/BF03353632